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An Analysis Method of Stresses Developed by Fill Soil ina Trench with Inclined Wall

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(1) Introduction

The use of buried conduits will increase in the future because of the preventing intermixture of sewage, the increasing effect of water utilization and so on. This depends upon the progressed soil enginnering and the economy of soil structures. Fortunately some results of the reseach are currently available and further there is an urgent demand for more accurate and more extensive design method.

It is well known that the magnitude of pressure on buried conduit in a trench may be different from the sum of the weight of the soil prisum and the full effects of a uniform static overpressure. Several authors have contributed to the development of the analytical methods for evaluating the loads produced by static overburden pressures and a series of results of theoretical and experimental work conducted by Marston, Spangler and associates at Iowa State University during the prriod 1908—1952 has been published. In this method, the major importance of the method of installation of the structure and the fill material is emphasized, and the effects of the various factors related to construction procedure are taken into account in the development. Also these analysis methods are based upon the fact that the magnitude of pressure on a conduit is a function not only of the weight of the soil prisum, but also of certain vertical shearing forces which may be generated within the soil overburden. In these methods, however, it is assumed that the distribution of pressure on a horizontal plane in soil prisum is uniform and shearing force act at the wall only. These assumptions are different from the conditions of the stresses in the soil prisum.

Therefore, this paper is devoted to the development of an approximate analysis method for taking account of the effect of the distribution of the normal stresses and shearing forces in fill soil prisum.

(2) Stress Equilibrium Equation and Boundary Conditions

When we assume that the soil is isotropic and homogeneous granular material, the Mohr-Coulomb failure theory will be considered to be a yield criterion of soil. On this basis the Mohr-Coulomb criterion for an axisymmetric case may be written as follows:

$$\left(\frac{\sigma_z+\sigma_x}{2}\right)^2+\tau_{zx}^2=\left(C\cos\phi+\frac{\sigma_z+\sigma_x}{2}\sin\phi\right)^2 \qquad (1)$$

where σ_x, σ_z : normal stress components parallel to x and z axes

τ_{zx} : shearing stress component in rectangular co-ordinates x and z axes

ϕ : internal friction angle of soil

C : cohesion of soil

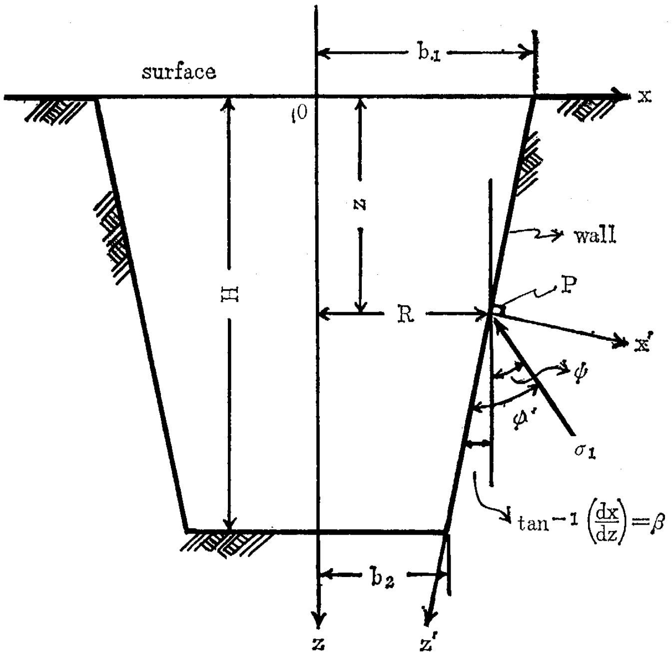
surface
b_1
x
0
z
H
wall
P
R
x'
φ
φ'
σ_1
$\tan^{-1}\left(\frac{dx}{dz}\right)=\beta$
b_2
z
z'

Fig 1.

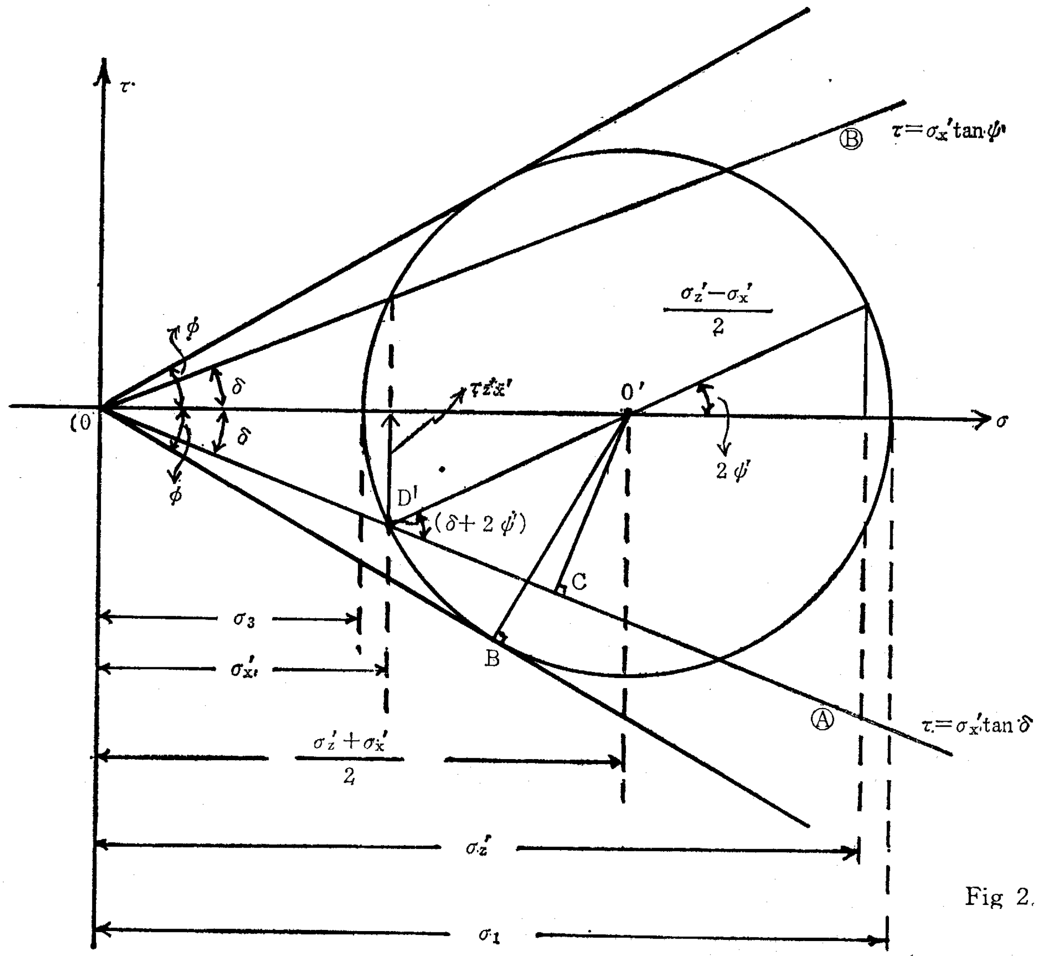
τ
Ⓑ
$\tau=\sigma_x'\tan\phi'$
$\frac{\sigma_z'-\sigma_x'}{2}$
φ
δ
δ
φ
0
$\tau z'x'$
0'
σ
2φ'
D'
(δ+2φ')
C
B
σ_3
σ_x'
Ⓐ
$\tau=\sigma_x'\tan\delta$
$\frac{\sigma_z'+\sigma_x'}{2}$
σ_z'
σ_1

Fig 2.

If the width and depth of a trench are sufficiently small compared with the length of it, the stresses developed in fill soil may be analyzed for two dimensional problem. In this case the differential equations of equilibrium of soil are represented by following forms:

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} = \gamma \quad (2)$$

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{zx}}{\partial z} = 0 \quad (3)$$

where γ: unit weight of soil

The boundary conditions at the wall of the trech are derived from Mohr's stress circle. If we neglect the cohesion of soil C, the failure stress conditions at the wall must be such as to satisfy the following relationship

$$\tau_{z'x'} = \sigma_{x'} \tan\delta \quad (4)$$

where δ is the friction angle between the soil and the wall. In fig. 1 a set of auxiliary wall co-ordinates z', x' at the point P are shown, and $\sigma_{z'}$, $\sigma_{x'}$, $\tau_{z'x'}$ are components of normal stress and shearing force in $z'-x'$ direction.

If the stresses at the wall are represented by Mohr's stress circle in fig. 2, then it is possible to impose the wall stress failure conditions of eq. (4). As the friction angle between the soil and the wall δ is generally smaller than ϕ, the line represented by eq. (4) must intersect the Mohr's stress circle and locates four points which could represent the wall stress conditions. We may understand easily these facts from lines A and B in fig. 2.

Since the soil mass will always act downwards in the vertical direction by gravity force, the shear stress at the wall will oppose impending motion of the fill soil. Consequently, major principal stress in this case act downwards in the vertical direction and the line A in fig. 2 is identified as that to be used for obtaining the stress conditions at the wall when failure occures.

In fig. 1 the angles ψ and ψ' are those which the major principal stress make with the z and z' axis respectively. At the wall ψ_R is given by following expression:

$$\psi_R = \psi'_R - \tan^{-1}\left(\frac{dx}{dz}\right) = \psi'_R - \beta \quad (5)$$

where $\beta = \tan^{-1}\left(\frac{dx}{dz}\right)$

Using the Mohr's stress circle, it is possible to obtain the relationships between the angles and stresses. In general we can write these as follows:

$$\sigma_z = \frac{\sigma_z + \sigma_x}{2}(1 + \sin\phi\cos 2\psi) \quad (6)$$

$$\sigma_x = \frac{\sigma_z + \sigma_x}{2}(1 - \sin\phi\cos 2\psi) \quad (7)$$

$$\tau_{zx} = \frac{\sigma_z + \sigma_x}{2}(\sin\phi\sin 2\psi) \quad (8)$$

In terms of the auxiliary co-ordinates in fig. 1 these expressions become:

$$\sigma_{z'} = \frac{\sigma_{z'} + \sigma_{x'}}{2}(1 + \sin\phi\cos 2\psi') \quad (9)$$

$$\sigma_{x'} = \frac{\sigma_{z'} + \sigma_{x'}}{2}(1 - \sin\phi\cos 2\psi') \quad (10)$$

$$\tau_{z'x'} = \frac{\sigma_{z'}+\sigma_{x'}}{2}\ (\sin\phi \sin 2\psi') \tag{11}$$

Considering the stress conditions at the wall, the value of ψ'_R is given by following expression

$$\psi'_R = \frac{1}{2}\left\{\sin^{-1}\left(\frac{\sin\delta}{\sin\phi}\right) - \delta\right\} \tag{12}$$

Substituting eq. (12) into eq. (5), we obtain the following expiession to determine ψ in terms of the $z-x$ co-ordinates:

$$\psi_R = \frac{1}{2}\left\{\sin^{-1}\left(\frac{\sin\delta}{\sin\phi}\right) - \delta\right\} - \beta \tag{13}$$

Using eqs. (6), (7) and (8), it is possible to indicate that the stresses at the wall are represented by the following forms in terms of the $z-x$ co-ordinates:

$$\sigma_z = \left(\frac{1+\sin\phi\cos 2\psi}{1-\sin\phi\cos 2\psi}\right)\sigma_x \tag{14}$$

$$\sigma_x = \left(\frac{1-\sin\phi\cos 2\psi}{1+\sin\phi\cos 2\psi}\right)\sigma_z \tag{15}$$

$$\tau_{zx} = \left(\frac{\sin\phi\sin 2\psi}{1-\sin\phi\cos 2\psi}\right)\sigma_x \tag{15'}$$

(3) Method of Approximate Solution

Let us consider now an approximate solution method for the differential equations of equilibrium of the fill soil in trench. To solve the simultaneous equations (1), (2) and (3) is very difficult and so it is shown in the present section that the method of integral relation may be applied to solve these equations.

From the results of elastic analysis and experiments of stresses in trenches, the vertical normalstresses may be assumed to vary in a parabolic manner across the width of a trench and the effects of the friction of the wall are considered to increase with decreasing the distance from the wall. Then the major principal stress at the wall must make an angle of ψ_R to the vertical line.

Now we consider this case by assuming that the approximate expression for the horizontal distribution of σ_z is given as follows:

$$\sigma_z = \gamma z + \left\{1 + \left(\frac{x}{R}\right)^2\right\} f(z) \qquad (0 \leqq x \leqq R) \tag{16}$$

where R : half width of trench at depth z

x : distance from z axis

$f(z)$: arbitrary function of z only

Denoting the width of the trench along the line $z=0$ and $z=H$ by b_1 and b_2, we have at any depth of the trench

$$R(z) = b_1 - \lambda z \tag{17}$$

where $\lambda = \dfrac{1}{H}(b_1 - b_2)$

This yields $R(z)=b_1$ at $z=0$, and b_2 at $z=H$. Consequently, the vertical normal stress in the fill soil is represented by

$$\sigma_z = \gamma z + \left\{1 + \left(\frac{x}{b_1 - \lambda z}\right)^2\right\} f(z) \qquad (0 \leqq x \leqq R,\ 0 \leqq z \leqq H) \tag{18}$$

Substituting eq. (18) into eq. (2) and integrating that equation with respect to x, we obtain following expression:

$$\tau_{zx} = -\left\{x+\frac{x^3}{3(b_1-\lambda z)^2}\right\}\frac{df(z)}{dz}-\frac{2\lambda x^3}{3(b_1-\lambda z)^3}f(z) \qquad (19)$$

Substituting eq. (19) into eq. (3) and integrating that equation with respect to x once more, we find the following expression:

$$\sigma_x = \left\{\frac{x^2}{2}+\frac{x^4}{12(b_1-\lambda z)^2}\right\}\frac{d^2f(z)}{dz^2}+\frac{\lambda x^4}{3(b_1-\lambda z)^3}\frac{df(z)}{dz}+\frac{\lambda^2 x^4}{(b_1-\lambda)z^4}f(z)+A(z) \qquad (20)$$

where $A(z)$ is arbitrary function z only.

For axisymmetric stress field the majoı principal stress at centre line will make an angle with z direction of zero, and the stress is vertical and $\psi=0$. At the centre line of the trench $x=0$, $\psi=0$ equation (15), therefore, becomes as follows:

$$\sigma_{x_0} = K\{\gamma z+f(z)\} \qquad (21)$$

where $K=\dfrac{1-\sin\phi}{1+\sin\phi}$

Using equations (15) and (20) for σ_{x0}, and substituting $x=0$, $\psi=0$ into these equations we can determine the arbitrary function $A(z)$.

$$A(z) = K\{\gamma z+f(z)\} \qquad (22)$$

Then the expression of σ_x becomes

$$\sigma_x = \left\{\frac{x^2}{2}+\frac{x^4}{12(b_1-\lambda z)^2}\right\}\frac{d^2f(z)}{dz^2}+\frac{\lambda x^4}{3(b_1-\lambda z)^3}\frac{df(z)}{dz}+\frac{\lambda x^4}{(b_1-\lambda z)^4}f(z)+K\{\gamma z+f(z)\} \qquad (23)$$

The value of σ_{xR} at the wall may be obtaind from eq. (15) and eq. (19) by substituting $x=R(z)=b_1-\lambda z$ and $\psi=\psi_R$ into these equations.

$$\sigma_{xR} = -\frac{4}{3}\mu(b_1-\lambda z)\frac{df(z)}{dz}-\frac{2}{3}\mu\lambda f(z) \qquad (24)$$

where $\mu=\dfrac{1-\sin\phi\cos 2\psi_R}{\sin\phi\sin 2\psi_R}$

Also, we can obtain the value of σ_{xR} at the wall by eq. (23).

Substituting $x=R(z)=b_1-\lambda z$ and $\psi=\psi_R$ into eq. (23), σ_{x_R} becomes

$$\sigma_{x_R} = \frac{7}{12}(b_1-\lambda z)^2\frac{d^2f(z)}{dz^2}+\frac{\lambda}{3}(b_1-\lambda z)\frac{df(z)}{dz}+\lambda f(z)+Kf(z)+K\gamma z \qquad (25)$$

From eq. (24) and eq. (25) we can get the differential equation to determine the function $f(z)$ contained in eq. (16).

$$(b_1-\lambda z)^2\frac{d^2f(z)}{dz^2}+a_1(b_1-\lambda z)\frac{df(z)}{dz}+a_2 f(z)=\frac{12}{7}K\gamma z \qquad (26)$$

where

$$a_1 = \frac{4}{7}(\lambda+4\mu)$$

$$a_2 = \frac{12}{7}\left(K+\lambda+\frac{2}{3}\lambda\mu\right)$$

This eq. (26) is the ordinary differential equ ion of Euler's type and it can be solved readily by using differential operator. Taking the following expression

$$(b_1 - \lambda z) = \exp t$$

the paticular solution of eq. (26) $f_0(z)$ is given by

$$f_0(z) = -\frac{12(m_1-m_2)}{7\lambda^2} K\gamma \left\{\left(\frac{1}{m_1{}^2}+\frac{1}{u_2{}^2}\right) + z\left(\frac{1}{m_1}+\frac{1}{m_2}\right)\right\} \quad (27)$$

where

$$m_1 = \frac{1}{2}\left\{\left(1-\frac{a_1}{\lambda^2}\right)+\sqrt{\left(1-\frac{a_1}{\lambda^2}\right)^2+\frac{4a_2}{\lambda^2}}\right\}$$

$$m_2 = \frac{1}{2}\left\{\left(1-\frac{a_1}{\lambda^2}\right)-\sqrt{\left(1-\frac{a_1}{\lambda^2}\right)^2+\frac{4a_2}{\lambda^2}}\right\}$$

Cosequently the solution of eq. (26) becomes

$$f(z) = C_1 e^{m_1 z} + C_2 e^{m_2 z}$$
$$-\frac{12(m_1-m_2)}{7\lambda^2} K\gamma \left\{\left(\frac{1}{m^{12}}+\frac{1}{m_2{}^2}\right) + z\left(\frac{1}{m_1}+\frac{1}{m_2}\right)\right\} \quad (28)$$

where C_1 and C_2 are integral constants.

Because the surface of the fill soil has no any loading, we can represent the boundry conditions at the surface as follows:

$$z=0 \quad : \quad \sigma_z=0, \qquad f(z)=0$$
$$z=0 \quad : \quad \frac{df(z)}{dz}=0 \quad (29)$$

The function $f(z)$ must be satisfied the boundry conditions and from eqs. (29) two constants in eq. (28) are found to be

$$C_1 = \frac{12K\gamma}{7\lambda^2}\,\frac{1}{m_1}\left(1-\frac{m_2}{m_1}\right)$$

$$C_2 = \frac{12K\gamma}{7\lambda^2}\,\frac{1}{m_2}\left(\frac{m_1}{m_2}-1\right) \quad (30)$$

Substituting these values of C_1 and C_2 in eq. (28), we can obtain the function $f(z)$ contained in eq. (18).

σ_x and τ_{zx} at any point of the fill soil can be found from eq. (23) and eq. (19) by using the equation of $f(z)$.

Rererences

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